

Testing QCD Sum Rule techniques on the lattice

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Abstract

Results for the first test of the “crude” QCD continuum model, commonly used in QCD Sum Rule analyses, are presented for baryon correlation functions. The QCD continuum model is found to effectively account for excited state contributions to the short-time regime of two-point correlation functions and allows the isolation of ground state properties. Confusion in the literature surrounding the physics represented in point-to-point correlation functions is also addressed. These results justify the use of the “crude” QCD continuum model and lend credence to the results of rigorous QCD Sum Rule analyses.

12.38.Gc, 12.38.Lg, 12.40.Yx

I. INTRODUCTION

Point-to-point correlation functions have been the subject of intense study since the early days of lattice field theory. In the quest for an *ab initio* determination of the low-lying hadron spectrum, lattice QCD investigations have focused on the large Euclidean-time tails of three-momentum-projected two-point correlation functions. As such, the short time regime, where excited state contributions are significant, has simply been discarded.

The lattice approach to QCD allows the determination of correlation functions deep in the nonperturbative regime. In contrast, the QCD Sum Rule (QCD-SR) method [1] is restricted to the near perturbative regime of the truncated Operator Product Expansion (OPE). In this regime, one cannot ignore the contributions of excited states in a phenomenological description of QCD correlation functions. To account for these contributions, the so-called “crude” QCD continuum model is introduced [1]. This model exploits the leading terms of the OPE, and introduces a sharp threshold marking the onset of the QCD continuum. The contributions of this model relative to the ground state, whose properties one is really trying to determine, are not small. They are typically 10 to 50% [2]. The validity of this model is relied upon to cleanly remove the excited state contaminations.

This investigation examines the physics in the near perturbative regime of point-to-point correlation functions where QCD-SR analyses are performed. The QCD continuum model is constructed for three-momentum-projected Euclidean-time two-point functions following the techniques established in the QCD-SR approach. This model will then be used as a probe of the physics represented in lattice QCD correlation functions, and as a test of QCD-SR techniques. This investigation explores some of the ideas briefly summarized in Ref. [3] in greater depth and detail.

Some attention has recently been given to the behavior of lattice point-to-point correlation functions in the near perturbative regime [4,5]. There the focus was on correlation functions in coordinate space. It was concluded that the QCD-SR-inspired continuum model was sufficient to describe the lattice correlation functions over the calculated range. Using

the entire lattice correlation function (including the deep nonperturbative regime), ground state masses were extracted and were found to agree with conventional lattice analyses.

Here the emphasis is on determining ground state properties by examining only the first few points of the lattice correlation function in the near perturbative regime. This is similar in spirit to QCD-SR analyses. In this manner, the validity of the QCD continuum model is rigorously tested. By extending the analysis interval of the correlation function deeper into the nonperturbative regime, the evolution of fitted ground state properties may be monitored. A sensitivity to the analysis interval in the extracted parameters would indicate a failure of the QCD-SR-inspired continuum model. Comparisons are made with conventional lattice results where ground state properties are simply extracted from the tails of the correlation functions. The importance of higher order terms of the OPE in the formulation of the QCD continuum model is also examined. These terms were not investigated in Ref. [4,5].

One would also like to evaluate whether these techniques are useful in analyzing other lattice QCD correlation functions. Such techniques may be of practical use for analyzing two-point correlation functions which become noisy prior to a clear ground state domination, such as heavy-light meson correlators. In a subsequent paper [6], these techniques will be exploited to investigate nucleon properties obtained from unconventional interpolating fields.

The outline of this paper is as follows. Section II introduces the definition of the two-point function examined in this investigation and summarizes the lattice techniques and parameters. In Section III, the QCD continuum model is derived for three-momentum-projected two-point correlation functions in Euclidean space. Issues associated with relating the lattice and continuum (lattice spacing $a \rightarrow 0$) formulations of the model are discussed here. Fits of the nucleon correlator are presented in Section IV. The validity of the QCD continuum model as implemented in the QCD Sum Rule approach is evaluated in Section V. The fit parameters are compared with other approaches in Section VI. In section VII, the physics represented in two-point correlation functions is discussed, where erroneous conclusions in the literature are addressed. Finally, Section VIII summarizes the findings of

this investigation.

II. LATTICE CORRELATION FUNCTIONS

A. Interpolating Fields

In lattice calculations, the commonly used interpolating field for the proton has the form

$$\chi_1(x) = \epsilon^{abc} \left(u^{Ta}(x) C \gamma_5 d^b(x) \right) u^c(x). \quad (2.1)$$

Here, we follow the notation of Sakurai [7]. The Dirac gamma matrices are Hermitian and satisfy $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$, with $\sigma_{\mu\nu} = \frac{1}{2i} [\gamma_\mu, \gamma_\nu]$. $C = \gamma_4\gamma_2$ is the charge conjugation matrix, a, b, c are color indices, $u(x)$ is a u -quark field, and the superscript T denotes transpose. Dirac indices have been suppressed.

In the QCD Sum Rule approach, it is common to find linear combinations of this interpolating field and

$$\chi_2(x) = \epsilon^{abc} \left(u^{Ta}(x) C d^b(x) \right) \gamma_5 u^c(x), \quad (2.2)$$

which vanishes in the nonrelativistic limit. With the use of Fierz relations, the combination of the above two interpolating fields with a relative minus sign may be written

$$\begin{aligned} \chi_{\text{SR}}(x) &= \epsilon^{abc} \left(u^{Ta}(x) C \gamma_\mu u^b(x) \right) \gamma_5 \gamma^\mu d^c(x), \\ &= 2(\chi_2 - \chi_1), \end{aligned} \quad (2.3)$$

giving the proton interpolating field often found in QCD Sum Rule calculations [2,8,9].

Here the task is to evaluate the validity of the QCD-SR-inspired continuum model. This issue is best addressed with the use of the interpolating field χ_1 of (2.1). For comparison with other calculations, results for χ_{SR} are also reported.

The Δ resonance is also considered for comparison with the nucleon results. For simplicity, the Δ^{++} interpolating field is used,

$$\chi_\Delta^\mu(x) = \epsilon^{abc} \left(u^{Ta}(x) C \gamma^\mu u^b(x) \right) u^c(x). \quad (2.4)$$

B. Two-Point Function

Two-point correlation functions are used to determine hadron masses. Consider the following two-point function for the nucleon,

$$G_2(t, \vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \text{tr} \left[\Gamma_4 \langle 0 | T \{ \chi_1(x) \bar{\chi}_1(0) \} | 0 \rangle \right]. \quad (2.5)$$

Here, $\Gamma_4 = (1 + \gamma_4)/4$ projects positive parity states for $\vec{p} = 0$, and tr indicates the trace over Dirac indices. Correlation functions at the quark level are obtained through the standard procedure of contracting out time-ordered pairs of quark field operators,

$$G_2(t, \vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \text{tr} \left[\Gamma_4 \epsilon^{abc} \epsilon^{a'b'c'} \left\{ S_u^{aa'}(x, 0) \text{tr} \left[S_u^{bb'}(x, 0) \tilde{C} S_d^{cc'T}(x, 0) \tilde{C}^{-1} \right] \right. \right. \\ \left. \left. + S_u^{aa'}(x, 0) \tilde{C} S_d^{cc'T}(x, 0) \tilde{C}^{-1} S_u^{bb'}(x, 0) \right\} \right], \quad (2.6)$$

where $\tilde{C} = C\gamma_5$, and $S_u^{aa'}(x, 0) = T \{ u^a(x), \bar{u}^{a'}(0) \}$, *etc.*

C. Lattice Techniques

Here we briefly summarize the lattice techniques used to calculate (2.6). Additional details may be found in Ref. [10]. Wilson's formulation is used for both the gauge and fermionic action. $SU(2)$ -isospin symmetry is enforced by equating the Wilson hopping parameters $\kappa_u = \kappa_d = \kappa$. Three values of κ are selected and are denoted $\kappa_1 = 0.152$, $\kappa_2 = 0.154$ and $\kappa_3 = 0.156$. To make contact with the physical world, the mass, continuum threshold and interpolating field coupling strengths calculated at the three values of κ are linearly extrapolated to $\kappa_{\text{cr}} = 0.1598(2)$ where an extrapolation of the squared pion mass vanishes. Differences between linear extrapolations to $m_\pi = 0$ as opposed to the physical pion mass are small and are neglected in the following.

Twenty-eight quenched gauge configurations were generated [10] by the Cabibbo-Marinari [11] pseudo-heat-bath method on a $24 \times 12 \times 12 \times 24$ periodic lattice at $\beta = 5.9$. Configurations were selected after 5000 thermalization sweeps from a cold start, and every 1000 sweeps thereafter [12].

Dirichlet boundary conditions are used for fermions in the time direction. Time slices are labeled from 1 to 24, with the δ -function source at $t = 4$. To minimize noise in the Green functions, the parity symmetry of the correlation functions, and the equal weighting of $\{U\}$ and $\{U^*\}$ gauge configurations in the lattice action are exploited. The nucleon mass determined from χ_1 of (2.1) is used to set the lattice spacing. This estimate lies between other estimates based on the string tension or the ρ -meson mass. The lattice spacing is determined to be $a = 0.132(4)$ fm, and $a^{-1} = 1.49(5)$ GeV.

Statistical uncertainties in the lattice correlation functions are estimated by a single elimination jackknife [13]. A covariance matrix fit of the pole plus QCD continuum model over a typical range of 15 time slices is likely to be unreliable for 28 gauge configurations [14]. Instead we use standard statistical error analysis in which correlations among the fit parameters are accounted for. The Gauss-Newton method is used to minimize χ^2 . Uncertainties are taken from the standard error ellipse [15] at $\chi^2 = \chi_{\min}^2 + 1$.

D. Mean-Field Improvement

In this analysis, the nucleon coupling strength (or residue of the nucleon pole) is determined in absolute terms, without resorting to a ratio of the QCD continuum contributions as done in [4,5]. Our approach requires the use of mean-field improvement [16,17]

$$\psi^{\text{Continuum}} \equiv \frac{\sqrt{2\kappa}}{a^{3/2}} \psi^{\text{Lattice}}, \quad (2.7)$$

where

$$\sqrt{2\kappa} \rightarrow \left(1 - \frac{3\kappa}{4\kappa_{\text{cr}}}\right)^{1/2}, \quad (2.8)$$

to account for otherwise large renormalization factors. While (2.8) is derived for heavy quarks [16], its use in light quark studies has already been suggested in [17]. The κ dependence of these two wave function normalizations is very different and use of the mean-field improved normalization is crucial to recovering the correct mass independence of the Wilson

coefficient of the identity operator. This point will be further illustrated in the following section.

The implementation of Wilson fermions on the lattice induces mixing between the composite nucleon interpolating fields of (2.1) and (2.2), reflecting the breaking of chiral symmetry [18]. This mixing is discussed in more detail in Ref. [6]. The important point is that the mixing of the interpolating fields χ_1 and χ_2 is negligible. As a result, it is possible to identify the properties of these interpolating fields determined on the lattice with those of their continuum ($a \rightarrow 0$) counterparts to a good approximation. The principle renormalization constant C_1^L has been determined in the mean-field approach [17] and is used in the following. The renormalization at the scale of $1/a$ is

$$\chi^{\text{Continuum}} = \frac{Z_{\chi N}}{a^{9/2}} \chi^{\text{Lattice}} \left(1 - \frac{3\kappa}{4\kappa_{\text{cr}}}\right)^{3/2}, \quad (2.9)$$

and $Z_{\chi N} = (1 - 0.73 \alpha_V) \simeq 0.80$ at $\beta = 5.9$.

III. THE EUCLIDEAN SPACE FORMULATION OF THE QCD CONTINUUM MODEL

A. Spectral Representation

At the phenomenological level, the two-point function is calculated by inserting a complete set of states N^i between the interpolators of (2.5) and defining

$$\langle 0 | \chi_1(0) | N^i, p, s \rangle = \lambda_1^i u(p, s). \quad (3.1)$$

Here, the coupling strength, λ_1^i , measures the ability of the interpolating field χ_1 to annihilate the i 'th nucleon excitation. For $\vec{p} = 0$ and Euclidean time $t \rightarrow \infty$, the ground state dominates and

$$G_2(t) \rightarrow \lambda_1^2 e^{-M_N t}. \quad (3.2)$$

We note here that the exponential suppression of excited states is somewhat similar to that encountered in Borel improved QCD-SR analyses. There the square of the mass appears

in the exponential as $\exp(-M_N^2 \tau^2)$ where τ is the inverse Borel parameter. The spectral representation is defined by

$$G_2(t) = \int_0^\infty \rho(s) e^{-st} ds, \quad (3.3)$$

and the spectral density is

$$\rho(s) = \lambda_1^2 \delta(s - M_N) + \zeta(s), \quad (3.4)$$

where $\zeta(s)$ provides the excited state contributions.

While it may be tempting to fit the correlation function in the shorter time regime by including additional poles in the spectral density,

$$\rho(s) = \lambda_N^2 \delta(s - M_N) + \lambda_{N^*}^2 \delta(s - M_{N^*}) + \dots, \quad (3.5)$$

such an approach fails in a number of ways. If one simply includes a couple of poles for the spectral density, one finds that the mass of the excitation is completely determined by the leading time slice that is considered in the fit. This is illustrated in Figure 1 where a fit of time slices 7 through 20 is shown. If time slice 6 is included, the mass of the second pole increases to accommodate the earlier time slice with smaller uncertainties.

One also finds that the coupling strength λ_{N^*} exceeds λ_N by an order of magnitude. Our physical intuition suggests the opposite. The coupling strength should decrease for excited states as these states are expected to have broader wave functions. The probability of finding three quarks at the same space time point is therefore smaller. In addition, such an approach is a poor representation of the physics believed to be there as it neglects the natural widths of the resonances and the QCD continuum of multiple hadron states. The correlation function is probably best described by many states of diminishing coupling strengths and increasing widths. It may be that the QCD-SR-inspired continuum model is an efficient way of characterizing this physics.

B. Operator Product Expansion (OPE)

The form of the spectral density used in the QCD continuum model is determined by the leading terms of the OPE surviving in the limit $t \rightarrow 0$. The calculation of these terms proceeds in the usual fashion of contracting out time-ordered pairs of quark field operators. For the nucleon interpolating field of (2.1) one has

$$G_2(t, \vec{p}=0) = \sum_{\vec{x}} \text{tr} \left[\Gamma_4 \epsilon^{abc} \epsilon^{a'b'c'} \left\{ S_u^{aa'} \text{tr} \left(S_u^{bb'} \gamma_5 C S_d^{Tcc'} C^{-1} \gamma_5 \right) + S_u^{aa'} \gamma_5 C S_d^{Tbb'} C^{-1} \gamma_5 S_u^{cc'} \right\} \right]. \quad (3.6)$$

In Euclidean space and coordinate gauge the quark propagator has the expansion

$$S_q^{aa'} = \frac{1}{2\pi^2} \frac{\gamma \cdot x}{x^4} \delta^{aa'} + \frac{1}{(2\pi)^2} \frac{m_q}{x^2} \delta^{aa'} - \frac{1}{2^2 3} \langle : \bar{q} q : \rangle \delta^{aa'} + \dots, \quad (3.7)$$

and $G_2(t)$ has the following OPE

$$G_2(t) \simeq \frac{3 \cdot 5^2}{2^8 \pi^4} \left(\frac{1}{t^6} + \frac{28}{25} \frac{m_q a}{t^5} + \frac{14}{25} \frac{m_q^2 a^2}{t^4} - \frac{56\pi^2}{75} \frac{\langle : \bar{q} q : \rangle a^3}{t^3} + \dots \right). \quad (3.8)$$

The quark mass used in the OPE is obtained from the standard relation

$$m_q a = \frac{1}{2} \left(\frac{1}{\kappa} - \frac{1}{\kappa_{\text{cr}}} \right), \quad (3.9)$$

and is renormalized to one loop [19,20].

C. QCD Continuum Model Contributions

The spectral density used in the QCD continuum model is defined by equating (3.3) and (3.8). Thus, $\rho(s)$ is given by the following Laplace transform pairs

$$\frac{1}{t^n} \rightarrow \frac{1}{(n-1)!} s^{n-1}, \quad n = 1, 2, 3, \dots, \quad \text{Re } t > 0. \quad (3.10)$$

The QCD continuum model is defined through the introduction of a threshold which marks the effective onset of excited states in the spectral density. This model is motivated by duality arguments as realized in the Q^2 dependence of the experimental cross section for $e^+e^- \rightarrow \text{hadrons}$. The QCD continuum model contribution is

$$\int_{s_0}^{\infty} \rho(s) e^{-st} ds = e^{-s_0 t} \sum_{n=1}^6 \sum_{k=0}^{n-1} \frac{1}{k!} \frac{s_0^k}{t^{n-k}} C_n \mathcal{O}_n, \quad (3.11)$$

where C_n and \mathcal{O}_n are the Wilson coefficient and normal ordered operator of the term t^{-n} in (3.8). The phenomenology of $G_2(t)$ is summarized by

$$G_2(t) = \lambda_1^2 e^{-M_N t} + \xi \int_{s_0}^{\infty} \rho(s) e^{-st} ds, \quad (3.12a)$$

$$\begin{aligned} &= \lambda_1^2 e^{-M_N t} + \xi \frac{3 \cdot 5^2}{(2^8 \pi^4)} e^{-s_0 t} \left(\right. \\ &\quad \left\{ \frac{1}{t^6} + \frac{s_0}{t^5} + \frac{1}{2} \frac{s_0^2}{t^4} + \frac{1}{6} \frac{s_0^3}{t^3} + \frac{1}{24} \frac{s_0^4}{t^2} + \frac{1}{120} \frac{s_0^5}{t} \right\} \\ &\quad + \frac{28 m_q a}{25} \left\{ \frac{1}{t^5} + \frac{s_0}{t^4} + \frac{1}{2} \frac{s_0^2}{t^3} + \frac{1}{6} \frac{s_0^3}{t^2} + \frac{1}{24} \frac{s_0^4}{t} \right\} \\ &\quad + \frac{14 m_q^2 a^2}{25} \left\{ \frac{1}{t^4} + \frac{s_0}{t^3} + \frac{1}{2} \frac{s_0^2}{t^2} + \frac{1}{6} \frac{s_0^3}{t} \right\} \\ &\quad \left. - \frac{56 \pi^2 \langle \bar{q} q \rangle a^3}{75} \left\{ \frac{1}{t^3} + \frac{s_0}{t^2} + \frac{1}{2} \frac{s_0^2}{t} \right\} + \dots \right). \end{aligned} \quad (3.12b)$$

Here we have introduced an additional parameter, ξ , governing the strength of the QCD continuum model. Strictly speaking, $\xi = 1$ in the continuum limit, $a \rightarrow 0$, but here is optimized with λ_1 , M_N , and s_0 to account for enhancement of the correlator in the short time regime due to lattice anisotropy. Its deviation from 1 is a reflection of the fact that we are matching the model formulated in the continuum limit to the lattice results at finite a . Ref. [5] found the anisotropy to be large for $x - x_0 < 6$ for free quark correlators and remain large in their interacting simulation at $\beta = 5.7$. At $\beta = 5.9$ there is some hope that anisotropy issues will be less problematic for the Fourier transformed correlators presented here. However, at very short times the quarks are essentially free and the anisotropy must be accommodated. With this approach, the effects of lattice anisotropy are absorbed through a combination of a larger QCD continuum model strength ($\xi > 1$) and marginally larger continuum threshold (s_0).

D. Lattice Cutoffs

Unlike coordinate space lattice analyses [4,5], infrared lattice artifacts are not a significant problem for this approach. The Fourier transform weight $\exp(-i\vec{p} \cdot \vec{x})$ is correct for all propagator paths including those which wrap around the lattice spatial dimensions.

The ultraviolet lattice cutoff may be modeled in a manner similar to that for the QCD continuum model. The upper limit of the integral over the spectral density of (3.12a) is cut off at Λ , and additional terms appear in (3.12b). As an example, consider the leading term of the OPE. In this case,

$$\frac{1}{t^6} = \int_0^\infty \frac{1}{5!} s^5 e^{-st} ds, \quad (3.13)$$

is modified to

$$\int_0^\Lambda \frac{1}{5!} s^5 e^{-st} ds = \frac{1}{t^6} - e^{-\Lambda t} \left\{ \frac{1}{t^6} + \frac{\Lambda}{t^5} + \frac{1}{2} \frac{\Lambda^2}{t^4} + \frac{1}{6} \frac{\Lambda^3}{t^3} + \frac{1}{24} \frac{\Lambda^4}{t^2} + \frac{1}{120} \frac{\Lambda^5}{t} \right\}. \quad (3.14)$$

Figure 2 illustrates the C_6/t^6 term with the ultraviolet cutoff correction (dashed line) fit to the lattice data from $t = 5 \rightarrow 7$ by optimizing Λ and C_6 . The corresponding curve for C_6/t^6 without the ultraviolet correction (solid line) is also indicated. The optimum value for Λ is 4.55(3) which is not too different from the momentum cutoff of π . Of course, the two numbers need not agree since we are modeling the ultraviolet cutoff and matching a continuum formulation, ($a \rightarrow 0$), to the lattice. The correction term accounting for the ultraviolet lattice cutoff is negligible by $t - t_0 = 2$. Rather than introducing an extra parameter in the fit, the correction is simply neglected, and all fits begin at $t = 6$ in the following.

A similar modification of the correlator due to the lattice regularization was not accounted for in Ref. [5], when comparing hadron correlators calculated on the lattice with free quark propagators, and correlators using continuum, ($a \rightarrow 0$), propagators. Inclusion of these effects in Fig. 3 of Ref. [5] would suppress the continuum curve at short distances such that it may continue to follow the diagonal elements of the lattice data more closely.

E. QCD Continuum Model Summary

Figure 3 displays the total and individual contributions to the QCD continuum model term of (3.12). For comparison, the total and individual contributions to the OPE of (3.8) are illustrated in Figure 4. The higher order terms of the OPE become significant at large time separations. Therefore, a truncated OPE will have a corresponding upper limit in time separations within which reasonable convergence is maintained. In contrast, the exponential of the continuum threshold in the QCD continuum model acts to suppress the higher order terms. The relative contributions of the subsequent terms of the OPE to the QCD continuum model are well ordered throughout the fitting regime, and inclusion of the first few terms of the OPE is adequate. Indeed, the contributions from the m_q^2 and quark-condensate terms make negligible contributions relative to the identity and m_q terms. These higher-order terms are not included in the following analysis. In summary, the phenomenological description of the correlation function is taken to be

$$G_2(t) = \lambda_1^2 e^{-M_N t} + \xi \frac{3 \cdot 5^2}{(2^8 \pi^4)} e^{-s_0 t} \left(\begin{aligned} &\left\{ \frac{1}{t^6} + \frac{s_0}{t^5} + \frac{1}{2} \frac{s_0^2}{t^4} + \frac{1}{6} \frac{s_0^3}{t^3} + \frac{1}{24} \frac{s_0^4}{t^2} + \frac{1}{120} \frac{s_0^5}{t} \right\} \\ &+ \frac{28 m_q a}{25} \left\{ \frac{1}{t^5} + \frac{s_0}{t^4} + \frac{1}{2} \frac{s_0^2}{t^3} + \frac{1}{6} \frac{s_0^3}{t^2} + \frac{1}{24} \frac{s_0^4}{t} \right\} \end{aligned} \right). \quad (3.15)$$

The shaded areas of Figure 4 indicate the regions typically excluded from Borel-transformed QCD-SR analyses. The remaining region is far from the regime where the ground state pole dominates the correlation function. A rigorous uncertainty analysis is needed to ascertain the predictive power of QCD Sum Rules [21].

IV. NUCLEON CORRELATOR FITS

Among the first of things to verify is the leading time dependence of $G_2(t)$. Here we consider the lattice data at $\kappa = 0.156$ where m_q corrections are smallest and one has the best chance to verify that the dependence is indeed proportional to $1/t^6$. Fitting time slices

$t = 6$ and $t = 7$ to the functional forms of t^{-7} , t^{-6} , and t^{-5} yields a χ^2/dof of 80, 4.8, and 230 respectively, suggesting the predominant behavior is $1/t^6$ as anticipated.

Our purpose is to test whether the nucleon mass and coupling strength can be obtained accurately from a fit considering only the first few points of the correlation function. The lattice correlation functions [10] are fit with (3.15) in a four-parameter search of λ_1 , M_N , s_0 and ξ in analysis intervals from $t = 6 \rightarrow t_f$ where t_f ranges from 11 through 23. Figure 5 illustrates these 13 fits of the lattice correlation function at the smallest value of κ considered. The statistical uncertainties are smallest for this quark mass and provides the best opportunity for revealing structure in the lattice correlation function that is not accounted for by the QCD continuum model. The typical χ^2/dof for these pole plus QCD continuum model fits is 0.6.

The nucleon mass determined in each of the intervals is plotted as a function of t_f in Figure 6. Pole plus QCD continuum model fits are compared with simple pole fits. Figure 7 illustrates similar results for the coupling strength. It is interesting to see that the simple pole determination of λ_1 fails to form a plateau at large time separations. Similar results are seen for the larger values of $\kappa = 0.154$ and 0.156 . The fit parameters extracted for the region $t = 6 \rightarrow 20$ are summarized in Table I [22]. The plateau in M_N and λ_1 for pole plus QCD continuum model fits indicates that the QCD continuum model effectively accounts for excited state contaminations in the correlation functions. The search parameters s_0 and ξ display a similar plateau as one might expect, since the regime in which these parameters are largely determined is common to all intervals.

Table I also indicates a value for ξ obtained by fitting $\xi \times$ the first two terms of the OPE to time slices $t = 6$ and 7 . Higher order terms not included in (3.8) are expected to be significant and therefore ξ obtained in this manner is only a rough estimate. Its similarity to that obtained from the QCD continuum model verifies the deviation of ξ from unity is a lattice artifact and not a failure of the QCD continuum model. It should also be mentioned that ξ has been defined with respect to the Wilson coefficient of the identity operator only to leading order in perturbation theory. α_s and leading log corrections are expected to reduce

ξ by about 25%.

A similar fit including the Wilson coefficient of the m_q correction, C_5 , as a search parameter confirms the OPE value for the coefficient ratio C_5/C_6 . The fact that this estimate agrees with the OPE prediction confirms the role of ξ as an overall continuum model strength. As such, ξ is expected to be independent of the quark mass in the same manner that C_6 , the coefficient of the identity operator, is. To demonstrate that this is in fact the case we present Figure 8. Here the quark mass dependence of ξ is illustrated (solid curve) for the three values of κ considered on the lattice as well as the value linearly extrapolated to κ_{cr} where the pion mass vanishes. ξ is nearly independent of the quark mass. This behavior is also in accord with the fact that enhancement of the correlator due to lattice anisotropy is largest at very short time separations where the OPE is dominated by the identity operator.

This result is not trivial, as it requires the use of the OPE value for C_5 and the use of mean-field-improved operators. If the naïve wave function normalization, $\sqrt{2\kappa}$, is used, ξ varies as indicated by the dashed line plotted in Figure 8.

In summary, the pole plus QCD continuum model allows the extraction of nucleon ground state properties, even for an interval as small as $t = 6 \rightarrow 11$. The techniques examined here may be useful in analyzing correlation functions that suffer a loss of signal prior to clear ground state domination. These techniques are used in a subsequent paper investigating nucleon properties obtained from unconventional interpolating fields [6].

V. QCD SUM RULE TEST

In the sum rule approach ξ is fixed to unity by the OPE. To test the QCD-SR method as closely as possible to its actual implementation, we will use the value for ξ obtained in the previous fit from $t = 6 \rightarrow 20$. This value is similar to that obtained by fitting (3.8) to the first few time slices of the lattice data.

Figure 9 illustrates the correlation function fits for the lightest quark mass at $\kappa = 0.156$ for the three-parameter search of λ_1 , M_N and s_0 , in analysis intervals from $t = 6 \rightarrow t_f$ where

t_f ranges from 9 through 23. While ideally we would like to start at $t_f = 7$, a minimum of 4 points is required for the fit with uncertainty estimates for the fit parameters. The typical χ^2/dof for these pole plus QCD continuum fits is 0.53. For the heaviest quark mass where the statistical uncertainties are smallest the typical χ^2/dof is 0.6.

Using a pole plus QCD continuum model, one can extract ground state nucleon properties using only the first few points of the correlation function. The stability of the nucleon mass to the analysis interval is displayed in Figure 10. The nucleon coupling strength is illustrated in Figure 11 where it is plotted as a function of the analysis interval. All three fit parameters are stable as functions of t_f . Moreover, the nucleon mass and coupling strength agree with the values extracted from the tail of the correlation function with a simple pole fit from $t_f - 7 \rightarrow t_f$ when $t_f \sim 20$ [23]. This indicates the QCD continuum model inspired by QCD Sum Rules is successful in quantitatively accounting for excited states in point-to-point correlation functions. However, interplay between the fit parameters gives rise to rather large uncertainties at $t_f = 9$.

The smallest analysis interval considered here is still somewhat generous relative to the interval considered in QCD-SR analyses. Borel improved Sum Rules including terms to dimension 8 in the OPE are typically analyzed in the regime $\tau = 1/M_{\text{Borel}} = 0.15 \rightarrow 0.35$ fm corresponding to $t \simeq 5 \rightarrow 7$ in the previous figures. However, Borel improved sum rules suppress excited states more effectively than the Euclidean formulation presented here, as the phenomenological side of Borel improved sum rules involves the square of the nucleon and excitation masses. The contribution of excited states for a given t_f is smaller for Borel improved Sum Rules and a direct comparison of the analysis intervals must take this into account.

One should also acknowledge the presence of an artificial enhancement of the lattice correlation functions at very short times. This requires the QCD continuum model to make larger contributions relative to the ground state. Any structure in the lattice correlation functions at short times reflecting structure in the spectral density is more prevalent in this analysis. Hence the enhancement of the correlator gives rise to a more demanding test of

the QCD continuum model. Structure not sufficiently accounted for by the model is more problematic here. The preceding results give a strong indication that the QCD-SR-inspired continuum model provides a sufficiently detailed description of the correlation functions in the short time regime to allow the extraction of ground state properties.

One might consider a similar analysis where the QCD continuum model is replaced by a second pole as in (3.5) with two poles. Such a comparison would demonstrate the importance of the QCD continuum model functional form and the sensitivity of the analysis presented here. We consider two fits of (3.5) to the lattice data. Since ξ was fixed in figures 10 and 11 to the optimal value obtained from a four-parameter fit to the time regime $6 \rightarrow 20$, we consider a three-parameter two-pole fit where λ_{N^*} is similarly fixed. We also present the results of a full four-parameter search for comparison.

For the two-pole spectral density of (3.5), both the ground state nucleon mass and coupling strength show a sensitivity to the analysis interval. These parameters do not form a plateau when plotted as a function of t_f . This failure of the single excitation hypothesis for the QCD continuum model is displayed in Figure 12 for the ground state mass and Figure 13 for the coupling strength.

Neither the mass nor the coupling strength extracted with a two-pole Ansatz agree with the values obtained from the single pole fits to the deep nonperturbative tail of the correlation function, where evolution in Euclidean time has isolated the ground state. A successful continuum model must reproduce the ground state properties obtained from the simple pole fits with $t_f \sim 20$ or 21 [23], as in Figures 10 and 11. For the two-pole spectral density, information on ground state properties has been sacrificed in accommodating the deficiencies of the spectral density in the short time regime, where the statistical uncertainties are relatively small. In this investigation, inclusion of the tail of the correlation function in the analysis interval is insufficient to determine the ground state properties when discrepancies made by a poor continuum model need to be accommodated.

In contrast, the QCD-Sum-Rule inspired continuum model produces ground state properties that agree with simple pole fits to the tail of the correlation function, and the fit

parameters are stable with respect to the analysis interval. Indeed this analysis is very sensitive to the continuum model Ansatz.

VI. COMPARISON WITH OTHER APPROACHES

A. Operator Product Expansions

Here we make a comparison of these techniques with those of other approaches for the nucleon and Δ channels. Since QCD Sum Rules and the Random Instanton Liquid Model (RILM) [24] typically use the interpolator χ_{SR} of (2.3) we repeat the previous analysis for this interpolator and the Δ interpolator. The OPEs for these interpolating fields are

$$G_{\text{SR}}(t) \simeq \frac{3 \cdot 5}{2^3 \pi^4} \left(\frac{1}{t^6} + \frac{2}{5} \frac{m_q a}{t^5} + \frac{1}{5} \frac{m_q^2 a^2}{t^4} - \frac{4\pi^2}{15} \frac{\langle : \bar{q} q : \rangle a^3}{t^3} + \dots \right), \quad (6.1)$$

and

$$\sum_{\mu=2,3} G_{\Delta}^{\mu\mu}(t) \simeq \frac{3^2 \cdot 5}{2^5 \pi^4} \left(\frac{1}{t^6} + \frac{4}{3} \frac{m_q a}{t^5} + \frac{2}{3} \frac{m_q^2 a^2}{t^4} - \frac{8\pi^2}{3} \frac{\langle : \bar{q} q : \rangle a^3}{t^3} + \dots \right), \quad (6.2)$$

where the $\mu = 2, 3$ Lorentz indices of the Δ interpolator have been summed over. Note that in this case the pole contribution to the phenomenological side of (6.2) is $(4/3)\lambda_{\Delta}^2 \exp(-M_{\Delta}t)$. The QCD continuum models are developed as outlined in Section III. A summary of the $t = 6 \rightarrow 20$ fit parameters for these interpolating fields is given in Tables II and III.

B. Systematic Uncertainties

Prior to comparing with other approaches, it is important to consider possible sources of systematic uncertainty, in addition to the statistical uncertainties indicated in the tables. As in most exploratory lattice calculations, this investigation considers a single value for the

coupling constant (or lattice spacing a) and a single lattice volume. As such, extrapolations to the infinite-volume continuum limit are not possible.

Naïvely, one might worry that finite volume errors are significant in this investigation. The lattice length in the shorter y and z directions is 1.58 fm and is small relative to the measured charge diameter, $2 \langle r^2 \rangle^{1/2}$, of 1.72(24) fm for the proton [25,26]. However, it is important to remember that the actual lattice simulations are performed with quark masses larger than the light current quark masses of a few MeV. For the quark masses considered on the lattice here, the proton charge diameter ranges from 1.0 to 1.2 fm. For estimating finite volume effects, the matter radius, $((2/3) \langle r_u^2 \rangle + (1/3) \langle r_d^2 \rangle)^{1/2}$, is a better measure. This diameter ranges from 1.0 fm to 1.1 fm, which is about 2/3 the length of the shortest dimension of the lattice.

Order a lattice spacing corrections to the lattice actions can also be a source of systematic error. Hadron spectrum calculations [27] indicate these errors are about 10% for hadron mass ratios at $\beta = 6.0$. The main quantity of interest here is the nucleon interpolating field coupling strength, λ_N . Since the nucleon mass is used to determine the lattice spacing, we have the advantage of determining the properties of a single hadron. Since M_N and λ_N are linked dynamically, and M_N is constrained to the physical value, we proceed under the assumption that finite volume and finite lattice spacing errors in λ_N are small relative to other sources of uncertainty. For the Δ one should expect systematic uncertainties from these effects to be the order of 10%. Quantification of these uncertainties remains a future endeavor of lattice QCD investigations.

The two most prominent sources of systematic uncertainty are the quenched approximation and the extrapolation of observables to the chiral limit. Our present understanding of the quenched approximation, obtained from numerical simulations of the full theory for a few observables, is that it is an excellent approximation to the full theory when the quark masses are sufficiently heavy. The quark masses considered in this investigation lie within this regime. Hence, it remains to estimate systematic uncertainty in the linear extrapolations of masses and coupling strengths.

Both M_N and λ_N are well behaved in the chiral limit. The leading nonanalytic terms of the chiral expansion are nonsingular. Our calculations of the nucleon mass indicate the quark mass dependence (or squared pion mass dependence) is linear to a good approximation. To order m_π^2 , the chiral expansion for M_N is linear in m_π^2 , and therefore our lattice determinations of the nucleon mass are linearly extrapolated to the chiral limit. Likewise, the more phenomenological continuum threshold is taken to have a similar mass dependence.

To order m_π^2 , the chiral expansion for λ_N is not linear in m_π^2 . The leading nonanalytic term is proportional to $m_\pi^2 \log m_\pi^2$. Here we use a phenomenological description of the m_π^2 behavior of λ_N to estimate the size of systematic error in linear extrapolations. For the interpolating field χ_{SR} it is possible to derive a simple expression for the pion mass dependence of λ_N [28],

$$\lambda_N = \lambda_0 \left(1 - \frac{3}{2^7 \pi^2} \frac{m_\pi^2}{f_\pi^2} \log \frac{m_\pi^2}{\Lambda^2} \right) + c m_\pi^2 + \dots . \quad (6.3)$$

Λ and c are redundant fit parameters. Here, Λ is fixed to the inverse lattice spacing.

Figure 14 illustrates the standard linear extrapolation and a fit of λ_0 and c in (6.3) to the lattice data. The linear extrapolation and the result from (6.3) differ by less than one standard deviation of the statistical uncertainty in the chiral limit. Hence an estimate of the combined statistical and systematic uncertainty for λ_N may be obtained by multiplying the statistical uncertainties in the tables by $\sqrt{2}$.

C. Comparison

The results summarized in Table IV for λ_{SR} evolved to a scale of 1.0 GeV^2 in the leading log approximation compare favorably with other approaches utilizing the pole plus QCD continuum phenomenology [29]. The correct nucleon sum rule predictions of [2,30] agree with the lattice results. Previous works addressing these issues [4,5,24] refer to the old and erroneous results of Ref. [8] and [31]. The numerous errors contained in Ref. [31] are discussed and corrected in Ref. [2]. The corrections are significant, and restore agreement with these

lattice results and with the RILM of [24]. The smaller value of λ_{SR} for Ref. [5] may be due to the omission of large α_s corrections to the Wilson coefficient of the identity operator used to normalize λ_{SR} . Such corrections could increase their result by approximately 20%.

There is some variation in the continuum threshold, s_0 from one approach to another. This is likely due to differences in the implementation of the QCD continuum models and correlation function enhancement in the short time regime reflecting lattice anisotropy. This enhancement is expected to induce a larger continuum threshold, and this is confirmed in Table IV.

Table V also displays uniformity among the residue of the pole for different approaches. Here the corrections to Ref. [31] are crucial [32]. Our analysis displays the usual suppression of M_Δ relative to M_N , typical of lattice analyses in which order a and finite volume corrections remain unaccounted [27]. The large value for M_Δ from Ref. [5] is mysterious. Once again the value of the continuum threshold is larger than in the QCD Sum Rule analysis as anticipated.

VII. THE PHYSICS OF INTERPOLATORS

There has been considerable confusion surrounding the physical significance of the baryon coupling strength λ , and more generally the physics represented in point-to-point correlation functions. Here we address these issues.

In Ref. [24] some conclusions are drawn from an incorrect interpretation of the physics represented in the interpolating field coupling strength, λ . From their study of *point-to-point* correlation functions, these authors claim “the octet and decuplet baryons have completely different wave functions” and that there is evidence of significant attraction in the scalar diquark channel. It must be stated that the phenomenological description of $G_2(t)$ in (3.12) involves the baryon mass, the continuum threshold describing the effective onset of excited states and λ , which indicates the ability of the particular interpolator to excite the baryon from the QCD vacuum. Information on the nucleon wave function that might be compared

with the Δ wave function is absent in point-to-point correlation functions.

While it is tempting to compare λ_N and λ_Δ in hopes of learning something about the distribution of quarks in baryons [24], such a comparison is not correct. It is important to recognize that λ reflects properties of *both* the baryon ground state *and* the interpolating field itself. Since the nucleon and Δ interpolating fields are different it should be no surprise to discover $\lambda_N \neq \lambda_\Delta$. Moreover, the nucleon interpolating field is not unique. It is possible to construct nucleon interpolators which yield values for λ_N both larger and smaller than λ_Δ . Moreover, their anomalous dimensions are different, making any comparison scale dependent. In short, there is little to be learned from a comparison of λ_N and λ_Δ .

Moreover, λ does not measure simply the wave function at the origin, but rather the wave function at the origin when

1. the three valence quarks by themselves are in a color singlet state;
2. the three valence quarks have the particular spin-flavor combination demanded by the interpolator.

The first criterion excludes the *majority* of the wave function including intermediate states of simple diagrams such as a single gluon exchange between quarks. Similarly, the second criterion excludes parts of the wave function that have no overlap with the interpolating field. These arguments also apply in general to wave function analyses where the locality of the baryon annihilation interpolator is relaxed and λ is determined as a function of quark field operator separations. Of course, wave function analyses also suffer from being either dependent on the gauge, or dependent on the path of link variables selected to implement gauge invariance.

The interpolating field acts to bias the physics represented in the wave function. For example, if an interpolator with predominant scalar diquark degrees of freedom is used to annihilate the baryon, then this part of the full baryon wave function will dominate the extracted wave function. The interpolating field filters out the part of the full wave function that looks like the interpolator. Thus, wave function analyses allow one to learn about

specific sectors of the complete wave function. However, it is dangerous to in turn attribute the properties obtained from a specific sector to those of the entire baryon wave function.

Many have expected it would be possible to find a perfect wave function for the creation of a hadron from the vacuum. A perfect smeared source would excite only the ground state hadron. Evolution in Euclidean time to suppress excited state contaminations would not be required. However, even the best smeared sources require a number of Euclidean time steps to isolate the ground state. This inability to obtain a perfect smeared source has a simple explanation. The wave function provides information on a particular spin-flavor-color Fock-space component of the full wave function. Without information on the full ground state wave function, a smeared operator will not be orthogonal to excited states and will always generate some excited state contaminations.

Of course, the only way to probe the properties of hadrons is through the use of a common interpolator or current such as the vector current, which has overlap with most hadrons. In this way, it is the properties of the hadrons themselves that give rise to differences in the extracted observables. For example, vector current matrix elements are determined via the calculation of three-point correlation functions in Ref. [10,33,34], where a variety of electromagnetic observables are reported. These results indicate that scalar diquark clustering in the nucleon is actually minimal in QCD [35,36].

VIII. CONCLUSIONS

A. The QCD Continuum Model

The physics represented in the short time regime of point-to-point correlation functions is described well by the QCD-Sum-Rule-inspired continuum model. The Laplace transform of the spectral density appears to be sufficient to render any structure in the spectral density insignificant in the short Euclidean time regime of point-to-point correlators. Similar conclusions are expected to hold for Borel-improved sum rules.

For the lattice correlation functions considered here, both the identity and m_q operators of the OPE are required when constructing QCD continuum models. Other higher order terms of the OPE make negligible contributions to the QCD continuum model due to the exponential suppression factor $\exp(-s_0 t)$. In general, consideration of the first few leading terms of a particular OPE should be sufficient for the construction of the QCD continuum model. The QCD continuum model is superior to the use of a second pole.

B. QCD Sum Rule Test

The QCD continuum model effectively accounts for excited state contributions to point-to-point correlation functions, allowing a determination of ground state properties in the short-time regime. However, interplay between pole and continuum model parameters leads to rather large uncertainties.

The smallest fit interval considered here is larger and extends deeper into the nonperturbative regime than that considered in QCD Sum Rule analyses. Hence it would be inappropriate to conclude that QCD Sum Rule techniques have been vindicated in this analysis. However, the success of the QCD continuum model here is encouraging. This analysis justifies the use of the “crude” QCD continuum model and lends credence to the results of rigorous QCD Sum Rule analyses.

C. Future Investigations

Having established that the QCD continuum model accounts for excited state contributions and allows the determination of ground state properties in the short-time regime, future lattice calculations may use this technique for analyzing correlation functions that suffer a loss of signal prior to a clear ground state domination. In a subsequent paper [6], these techniques will be exploited to investigate nucleon properties obtained from unconventional interpolating fields.

These techniques may be particularly helpful in the analysis of heavy-light meson correlators. Smeared-source to smeared-sink correlators suffer from large statistical uncertainties, while point-to-point correlators suffer from a loss of signal. One of these two correlators are needed in order to determine the meson decay constant. In addition, the mass splittings are relatively small in these systems, making isolation of the ground state by Euclidean time evolution inefficient. On the other hand, smeared-local correlators are well behaved and allow a clean determination of the ground state mass. Inclusion of the QCD continuum model in the phenomenological description of the correlator allows the analysis of the point-to-point correlator and provides a mechanism to account for the problematic excited state contaminations. With the pole position previously determined, a reliable extraction of the meson decay constant may be possible.

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TABLES

TABLE I. $\langle \chi_1 \bar{\chi}_1 \rangle$: Four-parameter search for the pole plus QCD continuum model.

Parameter	$\kappa_1 = 0.152$	$\kappa_2 = 0.154$	$\kappa_3 = 0.156$	$\kappa_{\text{cr}} = 0.1598(2)$
$M_N a$	1.109(8)	0.983(8)	0.858(8)	0.628(17) ^a
$\lambda_1 a^3 (\times 10^{-2})$	1.17(5)	0.94(4)	0.75(3)	0.38(7)
$s_0 a$	1.68(3)	1.58(3)	1.49(4)	1.32(7)
ξ	6.83(10)	6.74(9)	6.62(9)	6.42(19)
ξ from OPE fit	5.3(1)	5.6(1)	5.8(1)	

^aThe physical proton mass sets the lattice spacing $a = 0.132(4)$ fm.

TABLE II. $\frac{1}{4} \langle \chi_{\text{SR}} \bar{\chi}_{\text{SR}} \rangle$: Four-parameter search for the pole plus QCD continuum model.

Parameter	$\kappa_1 = 0.152$	$\kappa_2 = 0.154$	$\kappa_3 = 0.156$	$\kappa_{\text{cr}} = 0.1598(2)$
$M_N a$	1.106(8)	0.980(9)	0.863(9)	0.639(18)
$\lambda_1 a^3 (\times 10^{-2})$	1.14(5)	0.92(4)	0.76(3)	0.42(8)
$s_0 a$	1.63(3)	1.53(3)	1.45(4)	1.29(7)
ξ	5.46(7)	5.25(7)	5.02(7)	4.61(14)
ξ from OPE fit	4.5(5)	4.6(4)	4.6(3)	

TABLE III. $\langle \chi_\Delta \bar{\chi}_\Delta \rangle$: Four-parameter search for the pole plus QCD continuum model.

Parameter	$\kappa_1 = 0.152$	$\kappa_2 = 0.154$	$\kappa_3 = 0.156$	$\kappa_{\text{cr}} = 0.1598(2)$
$M_\Delta a$	1.164(8)	1.054(8)	0.953(9)	0.758(18)
$\lambda_\Delta a^3 (\times 10^{-2})$	2.91(11)	2.32(9)	1.82(8)	0.82(17)
$s_0 a$	1.84(2)	1.78(2)	1.73(3)	1.62(5)
ξ	10.3(1)	10.1(1)	10.0(1)	9.8(3)
ξ from OPE fit	7.0 \pm 1.5	7.3 \pm 1.3	7.6 \pm 1.2	

TABLE IV. Comparison with selected results for the nucleon channel $\langle \chi_{\text{SR}} \bar{\chi}_{\text{SR}} \rangle$.

Approach	Ref.	M_N	$\lambda_{\text{SR}}(1 \text{ GeV}^2)$	s_0
		(GeV)	(GeV ³)	(GeV)
This work		0.96(3)	0.027(5)	1.92(11)
QCD Sum Rules	Leinweber [2]	1.06(18)	0.031(6)	1.69(15)
Lattice (x -space)	Chu <i>et al.</i> [5]	0.95(5)	0.022(4)	<1.4
Instanton Liquid	Schäfer, <i>et al.</i> [24]	0.96(3)	0.032(1)	1.92(5)

 TABLE V. Comparison with selected results for the Δ resonance channel $\langle \chi_{\Delta} \bar{\chi}_{\Delta} \rangle$.

Approach	Reference	M_{Δ}	$\lambda_{\Delta}(1 \text{ GeV}^2)$	s_0
		(GeV)	(GeV ³)	(GeV)
This work		1.13(4)	0.024(6)	2.39(10)
QCD Sum Rules	Leinweber [32]	1.36	0.030	1.58
Lattice (x -space)	Chu <i>et al.</i> [5]	1.43(8)	0.037(6)	3.21(34)
Instanton Liquid	Schäfer, <i>et al.</i> [24]	1.44(7)	0.033(5)	1.96(10)

FIGURES

FIG. 1. A fit to the lattice correlation function (bullets) from $t = 7 \rightarrow 20$ utilizing a two pole Ansatz for the spectral density (solid curve). Individual pole contributions are illustrated by the dashed lines. When time slice 6 is included in the fit, the mass of the second pole increases to accommodate the earlier time slice.

FIG. 2. Comparison of the fit of the $1/t^6$ term with the ultraviolet cutoff correction (dashed line) of (3.14) and the corresponding curve for $1/t^6$ alone (solid line). The lattice cutoff correction term is negligible by $t - t_0 = 2$. In the following, the source and first point of the correlator are discarded and all fits begin at $t = 6$.

FIG. 3. QCD continuum model contributions. From top down the lines represent the total continuum model contribution, followed by the individual contributions of the identity operator, m_q correction, quark condensate and finally the m_q^2 correction.

FIG. 4. OPE contributions. From top down, at time slice 6, the lines represent the sum total of the first four terms of the OPE, followed by the individual contributions of the identity operator, m_q correction, quark condensate and finally the m_q^2 term. Line types are as in Figure 3. The crossing of the quark condensate contribution with the identity operator at $t = 8$ indicates the breakdown of the truncated OPE. The shaded areas indicate the regions typically excluded from Borel-transformed QCD Sum Rule analyses.

FIG. 5. The two-point correlator at $\kappa = 0.152$ for the nucleon interpolating field χ_1 of (2.1). The fits for the 12 analysis intervals are illustrated. The source position is at $t_0 = 4$. Neither the source nor $t = 5$ are included in the fit as indicated in the discussion surrounding Figure 2.

FIG. 6. The nucleon mass determined in each analysis interval plotted as a function of t_f . In this and the following figure, bullets correspond to pole plus QCD continuum fits from $t = 6 \rightarrow t_f$, and open squares, offset for clarity, illustrate a simple pole fit to the region $t_f - 7 \rightarrow t_f$ which was selected to give similar statistical uncertainties in the nucleon mass at $t = 20$.

FIG. 7. The nucleon coupling strength determined in each analysis interval plotted as a function of t_f . Symbols are as in Figure 6.

FIG. 8. The quark mass dependence of ξ for mean-field-improved wave function renormalization (solid curve) and naïve wave function normalization (dashed curve).

FIG. 9. Correlation function fits for each of the 15 analysis intervals $t = 6 \rightarrow t_f$, where $t_f = 9 \rightarrow 23$ for the QCD-SR method test.

FIG. 10. The nucleon mass determined in each analysis interval plotted as a function of t_f . In this and the following figure, bullets correspond to pole plus QCD continuum fits from $t = 6 \rightarrow t_f$, and open squares illustrate a simple pole fit to the region $t_f - 7 \rightarrow t_f$ as in Figure 6.

FIG. 11. The nucleon coupling strength determined in each analysis interval plotted as a function of t_f . Symbols are as in Figure 10.

FIG. 12. The nucleon ground state mass determined in each analysis interval, $t = 6 \rightarrow t_f$, plotted as a function of t_f for a spectral density involving two poles, as opposed to a pole plus QCD continuum model. Bullets correspond to a three-parameter fit with λ_{N^*} fixed as described in the text. Open circles illustrate the full four-parameter fit. The extracted mass depends on the analysis interval and fails to agree with the simple pole fits (open squares) from $t_f - 7 \rightarrow t_f$ at $t_f \sim 20$ or 21 . This confirms the sensitivity of this analysis to the QCD continuum model Ansatz.

FIG. 13. The nucleon coupling strength plotted as a function of t_f for a spectral density involving two poles. Symbols are as in Fig. 12. The dependence on the analysis interval and failure to agree with the simple pole fits (open squares) at $t_f \sim 20$ or 21 confirms the sensitivity of this analysis.

FIG. 14. Extrapolation of λ_{SR} to the chiral limit. The solid line indicates the standard linear extrapolation, while the dashed curve illustrates the two-parameter fit of (6.3) to the lattice data.